Monte Carlo Value at Risk
Monte Carlo VaR

Value at Risk (VaR) Definition

◆ The maximum likely loss on a portfolio for a given probability defined as $x\%$ confidence level over $N$ days

◆ $\Pr(\text{Loss} > \text{VaR}(x\%)) < 1 - x\%$
Value at Risk Roles

- Risk measurement
- Risk control
- Financial reporting
- Regulatory and economic capital
Monte Carlo VaR

Value at Risk Pros & Cons

◆ Pros
  ◆ Regulatory measurement for market risk
  ◆ Objective assessment
  ◆ Intuition and clear interpretation
  ◆ Consistent and flexible measurement

◆ Cons
  ◆ Doesn’t measure risk beyond the confidence level: tail risk
  ◆ Non sub-additive
Monte Carlo VaR

Three Value at Risk Approaches

- Parametric Value at Risk
- Historical Value at Risk
- Monte Carlo Value at Risk
Monte Carlo Value at Risk

**Assumption**

Assuming market factors follow certain stochastic processes.

**Pros**

- Easy back and stress test
- Good for high confidence level and tail risk

**Cons**

- Dependent on distribution assumption
- Calibration required
- Extensive computation
Monte Carlo VaR

Monte Carlo Value at Risk Methodology and Implementation

◆ Assume each market factor follows certain stochastic process: \( \vartheta(\sigma_i W_i) \) where \( W \) is a Wiener process
◆ Calibrate volatility \( \sigma_i \) for each market factor and pair-wise correlation \( \rho_{ij} \) for any two market factors
◆ Simulate market factor changes \( \delta_i \) based on the stochastic processes and correlated random variables.
◆ Generate market scenarios \( x_i = x_0 \delta_i \)
◆ Compute scenario PVs: \( P(x_i) \) and scenario P&L: \( P(x_i) - P(x_0) \)
◆ Sort all scenario P&Ls. The VaR is the number at 1% lowest level
Reference:

https://finpricing.com/lib/EqBarrier.html